

# Compositional Chaos and Musical Pleasure

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## Introduction

Mathematical concepts have played a significant role in the work of composers for generations. Composers as diverse as Dufay, Bartok, Schoenberg, Babbit, and Xenakis have applied various mathematical and numerological concepts to the composition of music. More recently, Austin, Dodge, Wuorinen, and others have investigated the compositional application of mathematical concepts derived from the emerging science of chaos (Dodge & Jerse, 1985). Concepts related to the study of chaos, including the study of fractional noise and fractal geometry, have arisen as the result of efforts by physicists, mathematicians, and other scientists to explain many of the apparently chaotic phenomena that appear in nature. Researchers of this newly evolving science of chaos have begun to explore such natural occurrences as the variations in sunspot patterns, under sea currents, global rainfall, the wobbling of the earth on its axis, and other anomalies of the physical world. As a result of this research, patterns have begun to emerge that, when plotted on a graph and mathematically analyzed, resemble a type of noise known as  $1/f$  (“one-over-f”).

## Purpose of Study

The purpose of this paper is: (1) to present a theoretical overview of  $1/f$  noise; (2) to provide an overview of research indicating that music which exhibits  $1/f$  characteristics produces music that is perceived as pleasurable by listeners; (3) to describe several algorithms used to generate  $1/f$  music; and (4) to describe a computer program I developed to facilitate the teaching of stochastic compositional methods using  $1/f$  noise and MIDI technology.

## $1/f$ in Music

In their article in *Nature*, Richard Voss and John Clark (1975) described the  $1/f$  audio power fluctuations contained in the recordings of compositions by Bach and Scott Joplin. Several years later, they analyzed works by Mario Davidovsky, Milton Babbit, Eliot Carter, Karlheinz Stockhausen, and Betsy Jolas. They discovered that these works also exhibited statistical characteristics consistent with  $1/f$  noise. These findings were obtained by passing recordings of the works through a band pass filter, squaring the voltages, and then passing the signal through a low pass filter with a cut-off frequency of 20 Hertz. The resultant low frequency voltage fluctuations were then subjected to an autocorrelation function to determine the relationship of these fluctuations over time (Voss & Clark, 1978).

They described how these voltage fluctuations evidenced the same correlations as those exhibited by  $1/f$  and related noise waveforms.  $1/f$  noise, or “flicker” noise, is a type of waveform that exhibits a high level of correlation over long time periods and can be described as having a power density spectrum of  $1/f^1$ . Another primary type of fractional noise, *white noise*, exhibits a power density spectrum of  $1/f^0$ ; this type of noise exhibits virtually no correlation over time. White noise, therefore, is said to have no “memory.” The third type of noise, *brown noise*, is highly correlated. The amplitude variations of its waveform resemble the “random” walk associated with Brownian motion. Brown noise exhibits a power density spectrum of  $1/f^2$ . Each event is highly dependent upon the value of the immediately preceding event, much like the walk of a drunken person stumbling from step to step.

## Noise Waveforms and Power Density Spectra

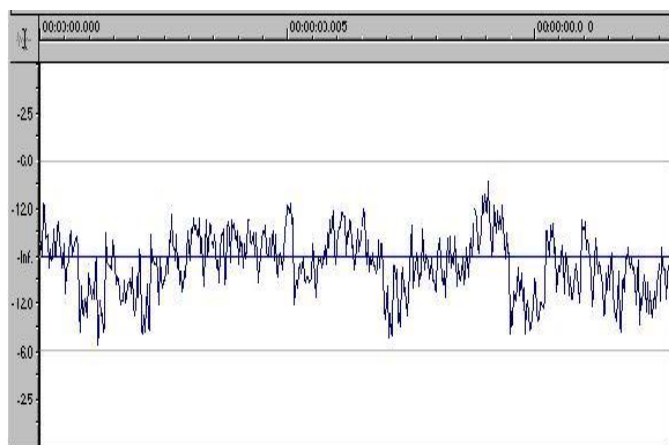
In order to understand the concept of  $1/f$  spectral density, it is helpful to first examine the basic anatomy of a waveform. When you observe a noise source on an oscilloscope, you see a graphic representation of the fluctuating amplitudes of the waveform. You can think of this waveform as the uppermost layer of this fractional noise phenomenon. In order to discern the  $1/f$  relationships themselves, one must look to more basic elements of the phenomenon, i.e., the amplitude of the spectral components that make up the waveform. All waveforms, other than

a simple sine wave, can be thought of as the interaction of a complex of sine waves of different amplitudes and frequencies. If we take a  $1/f$  noise waveform as depicted in **Figure 1** and perform a Fourier analysis of it, the  $1/f$  relationships of frequencies begin to emerge in the resulting frequency spectrum. The  $1/f$  relationships of these frequencies become fully evident when this spectrum is converted into a power density spectrum by squaring the amplitude of these frequency components;  $1/f^n$  expresses the manner in which the power density decreases with increasing frequency over a spectral range.

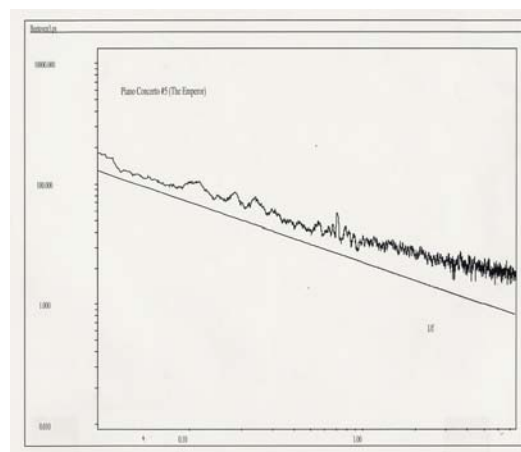
When viewing the power density spectrum of a white noise waveform, we see that the power density of the constituent frequencies is about equal. In this case there is no diminishing of power across the spectrum of white noise. However, when we look at the power spectrum of a  $1/f$  noise waveform, we see that the power density of the constituent frequencies tends to diminish as frequency increases. This decrease in power density can be expressed as  $1/f^1$ , which represents an attenuation rate of about three decibels per octave. When studying the spectrum of a brown noise waveform, we again see a diminishing of amplitude of each higher frequency similar to the  $1/f^1$  noise spectrum. This time, however, the spectrum falls off at a rate of approximately six decibels per octave ( $1/f^2$ ).

### Perception of Musical Pleasure

Voss and Clark hypothesized that since  $1/f$  noise sources contained the same time correlation as the music they had analyzed, it might be possible to use  $1/f$  noise to generate musical compositions that exhibited pleasing musical characteristics. They proceeded to generate melodies based upon the noise output of resistors, transistors, and filtered resistors, which provided white,  $1/f$ , and brown noise respectively. They used the minima and maxima of the resulting noise waveforms to determine the specific pitches and rhythmic values of those melodies. After a couple of years of subjective listening evaluations by hundreds of people at various universities and research institutions, it was evident that the  $1/f$  music was far more pleasing than the music generated by the uncorrelated white music or the highly correlated brown music.



**Figure 1**



**Figure 2**

In 1987, Charles Bennett, Ph.D., Professor of Physics at the University of North Carolina at Asheville, and I applied the same analytical techniques as used by Voss and Clark to ascertain the power density spectrum of a recording of Beethoven's Concerto No. 5 for Piano and Orchestra (Perahia, 1987). We sampled the entire recording and applied an autocorrelation function to the data. We hypothesized that because many listeners consider this work pleasurable, the spectrum would exhibit  $1/f$  characteristics. Our results supported this hypothesis, as shown by the power density spectrum graph shown in **Figure 2** (Bennett & Kirby, 1987).

In 1998, in an effort to quantify the human emotional response to music, Korean researchers analyzed the chaotic dynamics of electroencephalograms during human perception of  $1/f$  music. The results of their study supported the earlier findings of Voss and Clark. In their effort to study the universal emotional response to music, the Korean researchers generated a variety of musical stimuli based upon  $1/f$ , white, and brown noise waveforms. Their subjects rated the  $1/f$  music more pleasing than the brown or white music. The EEG results indicated that "brains which feel

more pleased show decreased chaotic electrophysiological behavior” (Jeong, Joung, & Kim, 1998). This study supports the notion that chaos is important to brain response, particularly as it relates to emotion.

### The Gardner Algorithms

In the Korean experiment, the researchers did not use the  $1/f$  noise emissions of electronic components to generate the noise waveforms to determine musical values, as did Voss and Clark. Instead, they used a simple process first described by Martin Gardner in *Scientific American* (1978). Gardner described how white music could be generated by creating a “spinner” that consisted of a spinning pointer and a circle divided into the notes of a scale. When spun, the spinner would randomly point to a note. Generating a pitch sequence was then a simple matter of sequentially spinning the device and notating the pitches. The same could be done for rhythmic values.

Brown music could be generated utilizing a similar spinner. In this case, the individual pitches were replaced by values of 0, +1, +2, +3, -1, -2, and -3. After selecting a starting pitch, the user would spin the pointer. Each of the values indicated the amount of rise, fall or lack of change that would occur in the sequence of notes. This process yielded a “random walk” kind of melodic movement akin to the Brownian motion reflected in the “wandering” fluctuations of a brown noise waveform. In this case, the high level of correlation between notes was evidenced by the dependence of each value’s influence upon succeeding values.

$1/f$  music required a slightly more complex algorithm. Gardner described Voss’s suggestion for a simplified procedure to produce music halfway between white and brown. A sequence of numbers would be written in binary notation so that their digits were lined up in columns with the lowest number at the top. Each column was then assigned a color corresponding to one of three colored dice. Using these three color-coded dice, one would throw all three dice and note the sum of the values shown on their faces. A pitch from a scale was assigned to that first value. Subsequent pitch values were then determined by throwing the die or dice that corresponded to a change or changes in binary digits in each of the columns. Each time this occurred, the sum of all the dice was noted and assigned a corresponding pitch. In their *Byte* article of 1986, Curtis Bahn and Charles Dodge described their adaptation of the Gardner-Voss method for generating  $1/f$  sequences of pitches and rhythms using the BASIC language and a Yamaha CX5-M music computer (Dodge & Bahn, 1986).

### Overview of the Algorithmic Composition Toolbox

In 1987, I began programming the *Algorithmic Composition Toolbox* (Kirby, 1988). This computer program uses MIDI protocol to generate compositions based on fractional noise. The toolbox works by synthesizing  $1/f$  waveforms, then assigning user-input pitches, velocities, durations, and other musical parameters to the fluctuating amplitudes of the waveform.

The white music algorithm sequences musical events in a random pattern or sequence; there is virtually no correlation among events. This is accomplished by simulating the throw of a die with the number of virtual sides dictated by the number of pitches, note or rest durations, velocities (dynamics), program changes, whole sequences, or other MIDI parameters contained in a user-determined *look-up table*. In **Figure 3**, a scale has been assigned to a range of numbers that can be called by the computer’s random number generator. To limit the range of numbers, a modulus value corresponding to the desired number of values is applied to the output of the random number generator. The random number function is first seeded with the computer’s clock time in order to assure that a different sequence is generated each time the random function is called.

The brown music algorithm causes events to be sequenced in a highly correlated manner. As noted earlier, sequences generated using  $1/f^2$  generally move in small steps, providing a kind of “random walk” effect. My algorithm automatically begins with the middle entry in the look-up table for its initial value. This value, and all subsequent ones, is each then increased, decreased or remains unchanged by adding values returned by the random number generator.

Sequences produced by the  $1/f$  music algorithm are moderately correlated when compared to white and brown sequences. **Figure 4** illustrates the algorithmic approach I adapted from the Gardner-Voss process for generating  $1/f$

sequences of pitches. In this example, I simulated the throwing of five imaginary six-sided dice. The modulus value corresponded to the total number of pitches in the pitch look-up table. The number of virtual “sides” of the dice and the number of simulated dice were dictated by the number of pitches in the look-up table that had been selected for a particular sequence. In this illustration, the five dice generated a sequence of 32 (2<sup>5</sup>) notes (numbered 0-31) from a scale of 26 possible notes. As dictated by the Gardner-Voss dice-throwing method, each time a digit in a column changed, a dice throw was simulated. The resulting number was added to the values already in place from preceding “throws.” Each resulting sum corresponded to a MIDI note number. As you can see, the most significant bit changed at a much lower rate than the lesser significant bits. In order to generate long sequences, the method was repeated, i.e., the sequence of binary numbers was repeated until the requisite number of pitches was generated. Examples of musical sequences generated by the toolbox are available online at <http://www.seriouscomposer.com/>.

MIDI Note Assignments: C Harmonic Minor Scale	
if 5 then MIDI note # 48 -> C3	
if 6 then MIDI note # 50 -> D3	
if 7 then MIDI note # 51 -> Eb3	
if 8 then MIDI note # 53 -> F3	
if 9 then MIDI note # 55 -> G3	
if 10 then MIDI note # 56 -> Ab3	
if 11 then MIDI note # 59 -> B3	
if 12 then MIDI note # 60 -> C4	
if 13 then MIDI note # 62 -> D4	
if 14 then MIDI note # 63 -> Eb4	
if 15 then MIDI note # 65 -> F4	
if 16 then MIDI note # 67 -> G4	
if 17 then MIDI note # 68 -> Ab4	
if 18 then MIDI note # 71 -> B4	
if 19 then MIDI note # 72 -> C5	
if 20 then MIDI note # 74 -> D5	
if 21 then MIDI note # 75 -> Eb5	
if 22 then MIDI note # 77 -> F5	
if 23 then MIDI note # 79 -> G5	
if 24 then MIDI note # 80 -> Ab5	
if 25 then MIDI note # 83 -> B5	
if 26 then MIDI note # 84 -> C6	
if 27 then MIDI note # 86 -> D6	
if 28 then MIDI note # 87 -> Eb6	
if 29 then MIDI note # 89 -> F6	
if 30 then MIDI note # 91 -> G6	

Figure 3

	Binary	5 (MSB)	4	3	2	1 (LSB)	Sum	MIDI Note #	Pitch
0	00000	5	2	3	4	4	18	71	B4
1	00001	(5)	(2)	(3)	(4)	3	17	68	Ab4
2	00010	(5)	(2)	(3)	3	4	17	68	Ab4
3	00011	(5)	(2)	(3)	(3)	2	15	65	F4
4	00100	etc.	(2)	4	2	6	19	72	C5
5	00101		(2)	(4)	(2)	4	17	68	Ab4
6	00110		(2)	(4)	4	3	18	71	B4
7	00111		(2)	(4)	(4)	4	19	72	C5
8	01000		5	5	6	2	23	79	G5
9	01001		(5)	(5)	(6)	4	25	83	B5
10	01010		etc.	etc.	3	6	24	80	Ab5
11	01011					3	21	75	Eb5
12	01100			5	2	4	21	75	Eb5
13	01101				6	1	22	77	F5
14	01110				1	3	19	72	C5
15	01111					6	22	77	F5
16	10000	3	6	2	3	1	15	65	F4
17	10001					4	18	71	B4
18	10010				5	2	18	71	B4
19	10011					6	22	77	F5
20	10100			5	2	4	20	74	D5
21	10101					5	21	75	Eb5
22	10110				4	2	20	74	D5
23	10111					6	24	80	Ab5
24	11000		1	6	3	2	15	65	F4
25	11001					5	18	71	B4
26	11010				2	3	15	65	F4
27	11011					1	13	62	D4
28	11100			5	3	5	17	68	Ab4
29	11101					4	16	67	G4
30	11110				1	6	16	67	G4
31	11111					5	15	65	F4
1a	00000	4	5	1	4	3	17	68	Ab4
2a	00001					4	18	71	B4
3a	00010				2	6	18	71	B4
Etc.	Etc.							Etc.	Etc.

Figure 4

In an attempt to reach beyond the simple micro-level applications of fractional noise concepts to the control of pitch, duration, and velocity, I also investigated the application of these organizational principles on a macro level. The program is designed to sequence and insert patch assignments in order to automatically orchestrate compositions according to 1/f patterns. Complex musical gestures can be algorithmically created or recorded in real time via a MIDI controller then manipulated as individual musical elements by means of the algorithms. The capability to alternately input data from multiple track files allows temporal distribution of musical events, e.g., an algorithmically sequenced collection of musical gestures whose occurrences are separated by silent intervals (contained in a separate file) of lengths determined by the algorithms. In order to reduce the oftentimes mechanistic sounding results of pitch sequences generated by the computer from source files input via computer keyboard, the program offers the capability of creating pitch, duration, and velocity files by performing them into the computer.

Using a MIDI controller, parameters can be recorded and then automatically stored in separate pitch, duration, and velocity files, thereby maintaining the subtle nuances of human performance. Additionally, sequences of individual parameters can be extracted from previously composed sequences. This feature allows a single pitch, duration, or velocity sequence to be used in conjunction with other sequences.

### **Aliquando Fidelis**

In an effort to explore the compositional potential of fractional noise, I composed over a dozen pieces using the toolbox. My first compositional experiment with the toolbox was a string quartet funded by the National Endowment for the Arts and the North Carolina Arts Council. Entitled *Aliquando Fidelis* (Kirby, 1989), which is Latin for “sometimes faithful,” the banks of pitches available to the computer included three scales. The first and third movements were derived from a harmonic minor scale; the second movement was based on a major scale; and the fourth movement was based upon a whole-tone scale. I intuitively determined the musical dynamics. The banks of rhythmic values were extracted from some of Beethoven’s works and entered into the computer as numeric values. While the computer had complete control over the selection of pitches, and no control over dynamics, it had varying degrees of control over the distribution of rhythmic values—“sometimes faithful” to Beethoven—sometimes not. Each individual instrumental part was assigned to a specific range of notes in order to prohibit the crossing of voices. Multiple versions of each part were generated and combined with others until I felt that the interrelationship of the instrumental lines imparted an intuitive feeling of musical sense. Excerpts from this work are available online at <http://www.seriouscomposer.com/>.

### **Conclusion**

Though pleasurable music appears to often contain  $1/f$  characteristics—and simple  $1/f$  sequences of pitches and durations seem to generate pleasurable musical experiences—it is not currently known if complex  $1/f$  music compositions yield the same kind of statistical content or psychological effects. Though many of the works previously analyzed using the autocorrelation method exhibited  $1/f$  characteristics, further investigation is needed to determine whether or not complex musical compositions generated with  $1/f$  algorithms provide the same kind of power density spectra, objectively measured brain activity, and psychological responses.

### **References**

- Bennett, C. & Kirby, W. (1987). Power density spectrum analysis of Beethoven’s Piano Concerto No. 5 for Piano and Orchestra, Op. 73, realized in the physics laboratory of the University of North Carolina at Asheville.
- Dodge, C. & Bahn, C. (1986, June). Musical Fractals. *Byte*, 185-196.
- Dodge, C. & Jerse, T. (1985). *Computer Music: Synthesis, Composition, and Performance*. New York, NY: Schirmer Books.
- Gardner, M. (1978). Mathematical Games: White and brown music, fractal curves and one-over-f fluctuations. *Scientific American*, 4, 16-32.
- Jeong, J., Joung, M. K., & Kim, S.Y. (1998) Quantification of emotion by nonlinear analysis of the chaotic dynamics of electroencephalograms during perception of  $1/f$ . *Biological Cybernetics*, 78, 217-225.
- Kirby, W. (1988). *Algorithmic Composition Toolbox* Available online from Serious Composer, Inc. Web site: <http://www.seriouscomposer.com/>
- Kirby, W. (1989). *Aliquando Fidelis for String Quartet*. Available online from Serious Composer, Inc. Web site: <http://www.seriouscomposer.com/>
- Perahia, M. (1987). Beethoven Concerto No. 5 ‘Emperor.’ [CD]. New York, NY: CBS MK 42330.
- Voss, R. F. and Clarke, J. (1975) ‘ $1/f$  noise’ in music and speech. *Nature* 258, 317-318.
- Voss, R. F. and Clarke, J. (1978).  $1/f$  noise in music: Music from  $1/f$  noise. *Journal of the Acoustic Society of America* 63(1), (258-263).